Acıbadem
University

FINAL EXAM
Spring 2023

Name (Please Print): _____

ACU Student Identification Number: _____

Department: Circle one: CSE or BME/MEG or MBG

Mat 102/112
Mathematics II / Calculus II

Instructor: Elçim Elgün Kırımlı

Term: Spring 2023

Number of exam pages: 8 (*including cover page*)

Date: 15 June, 2023

Time: 9.30 a.m. to 11.00 a.m.

Exam type: closed book and open ended

Additional Material Allowed: 1 page note sheet is allowed

~~Second side of each exam paper~~ **is for rough work calculations or additional space.**
Last page

Instructions

1. Write your name and ID number at the top of this page. Identify yourself on *every* page.
2. This exam consists of 6 questions and there are 100 points available. It will be graded out of 90 points
3. This exam contains 8 pages including the cover page. Check that you have them all.
4. Answer all questions in the spaces provided, using the backs of pages for overflow or rough work. The last page may also be used for rough work.
5. Show all your work required to obtain your answers.
6. **Calculators are not allowed.**

FOR EXAMINERS' USE ONLY			
Q	Mark	Q	Mark
1	/20	4	/15
2	/20	5	/16
3	/14	6	/15
Total			/90

1. This question contains 2 parts:

PART I: True/False questions. Determine whether the following statements are True or False. No explanation is required.

[2] (a) If $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^n$ and $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$, then $\vec{b} = \vec{c}$.

[2] (b) If $\sum a_k$ converges, then $\sum |a_k|$ also converges.

[2] (c) If $\vec{a} = (3, -4)$ and $\vec{u} = (2, 0)$, then $proj_{\vec{u}}\vec{a} = (3, 0)$.

[2] (d) The point $Q(1, 2, -5)$ lies on the plane $x - y + z + 6 = 0$.

PART II: Short Answer questions. Fill in the following blanks. No explanation is required.

[3] e) If a particle is moving with position function $\vec{r}(t) = t\vec{e_1} + t^2\vec{e_2}$, then the velocity at $t = 2$ is .....

[3] f) Let γ be parameterized by $\vec{r}(t) = (t\cos t, t\sin t, t)$ in Cartesian coordinates. In cylindrical coordinates, γ is given by .....

[3] g) If $\vec{a} = (3, 1)$ and $\vec{b} = (-1, 2)$, then $\|\vec{a} + 2\vec{b}\| =$ .....

[3] h) If $\vec{r}(t) = e^{-t^2}(1, 2, 1)$, then $\lim_{t\to\infty} \vec{r}(t) =$ .....

2. We will evaluate the integral $I = \int \frac{\sqrt{1-x^2}}{x} dx$:

[5] (a) Apply trigonometric substitution with $x = \sin u$.

[5] (b) Multiply and divide the integrand by $\sin u$, then apply change of variables with $t = \cos u$.

[5] (c) Apply partial fraction decomposition.

[5] (d) Evaluate the obtained integrals.

3. Determine whether the following series diverge or converge? For each alternating series, also determine type of convergence.

[4] (a) $\sum_{n=1}^{\infty} \frac{n}{2n+3}$

[4] (b) $\sum_{n=1}^{\infty} \frac{1}{n^2-1}$

[6] (c) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n^2}}$

[5] 4. (a) Find the radius and interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{2^{2n}}{n^2}(x-2)^n$.

[5] (b) Find a power series representation of the function $f(x) = \frac{1}{3-x}$ and determine its radius of convergence.

[5] (c) Find a power series representation of the function $f(x) = \ln|3-x|$ and determine its radius of convergence.

[16] 5. Use Chain rule to calculate $\frac{\partial F}{\partial s}$ and $\frac{\partial F}{\partial t}$ for the function $F(s,t) = f(x(s,t), y(s,t))$, where

$$f(x,y) = 2x^2y - xy^2 \qquad x(s,t) = t^2 + 3s^2 \quad \text{and} \quad y(s,t) = 3st$$

6. Consider the function $f(x,y) = \frac{x^2-y^2}{x^2+y^2}$.

[5] (a) Evaluate the limit $\lim_{(x,y)\to(0,0)} f(x,y)$. (Hint consider $x = 0, y = 0$ and $x = y$)

[5] (b) Find the gradient of f.

[5] (c) Determine the directional derivative of f in the direction of $\vec{u}(1,-2)$.

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