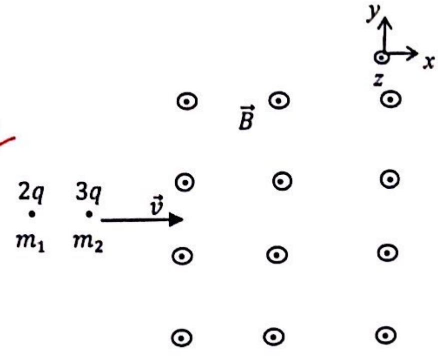
1- a) Two particles with masses m_1 and m_2 and charges $2q$ and $3q$ travel with the same velocity $\vec{v}$. They enter a region of uniform magnetic field of strength B at the same point as shown in the figure. In the magnetic field, they move in semicircles with radii $r_1 = R$ and $r_2 = 3R$.

i- Calculate the magnetic force on each charge when they just enter the field region. **(4 pts)**



$\vec{F}_1 = q_1 \vec{v} \times \vec{B}$

$\vec{F}_1 = 2q\, v\hat{i} \times B\hat{k} = 2qvB(-\hat{j})$ 2

$\vec{F}_2 = q_2 \vec{v} \times \vec{B} = 3qv\hat{i} \times B\hat{k}$

$\vec{F}_2 = 3qvB(-\hat{j})$ 2

ii- Calculate the ratio of their masses, $\frac{m_1}{m_2}$. **(4 pts)**

$qvB = \frac{mv^2}{R} \Rightarrow R = \frac{mv}{qB}$, $r_1 = R = \frac{m_1 v}{2qB}$ 2

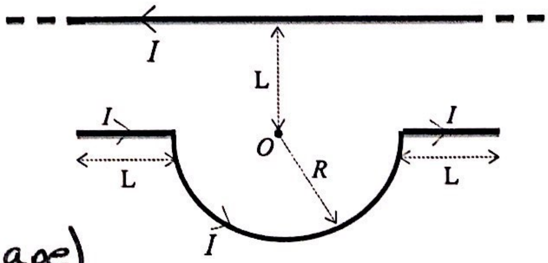
$r_2 = 3R = \frac{m_2 v}{3qB}$

$\frac{1}{3} = \frac{3m_1}{2m_2}$

$\frac{m_1}{m_2} = \frac{2}{9}$ 2

b) Calculate the net magnetic field $\vec{B}$ (both in magnitude and direction) at point O, due to the currents I in both long, straight wire and the semi-circular wire, shown in the figure. **(12 pts)**

(you must __derive__ the equations of the magnetic fields using Ampere's Law and Biot-Savart Law!)



$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$ 2

$B_1 2\pi L = \mu_0 I$

$\vec{B}_1 = \frac{\mu_0 I}{2\pi L} \hat{k}$ (out of page) 2

$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2}$ 2

$B_2 = \int dB_2 = \frac{\mu_0 I}{4\pi R^2} \int dl = \frac{\mu_0 I}{4\pi R^2} \frac{2\pi R}{2} = \frac{\mu_0 I}{4R}$ 2

$\vec{B}_2 = \frac{\mu_0 I}{4R} \hat{k}$ (out of page) $\quad \vec{B} = \vec{B}_1 + \vec{B}_2 = \left(\frac{\mu_0 I}{2\pi L} + \frac{\mu_0 I}{4R}\right)\hat{k}$ 4

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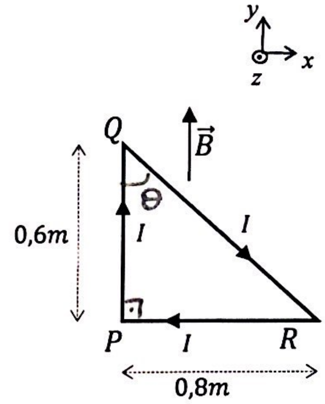
2- The triangular loop of wire shown in the figure carries a current $I = 8\ A$ in the direction shown. The loop is in a uniform magnetic field that has magnitude of $B = 5\ T$ and the direction which has the same direction as the current direction along PQ of the loop.

a) Find the magnetic force $\vec{F}$ (both in magnitude and direction) exerted by $\vec{B}$. **(12 pts)**

on *PQ*:



$$\vec{F} = I\vec{L} \times \vec{B} = IL\hat{j} \times B\hat{j} = 0$$

4

on *QR*:

$$\vec{F} = I(-L\cos\theta\hat{j} + L\sin\theta\hat{i}) \times B\hat{j}$$

$$= -ILB\frac{0.6}{1}(\hat{j}\times\hat{j}) + ILB\frac{0.8}{1}(\hat{i}\times\hat{j}) = ILB\,0.8\,\hat{k}$$

$$= 8\times1\times5\times0.8\,\hat{k}$$

$$= 32N\hat{k}$$

4

on *RP*:

$$\vec{F} = I(-L\hat{i}) \times B\hat{j} = -ILB\hat{k}$$

$$= -8\times0.8\times5\,\hat{k}$$

$$= -32\,\hat{k}\ N$$

4

b) What is the net magnetic force on the loop? **(3 pts)**

$$F_{net} = 0$$

3

c) The loop is rotated about an axis along side PR. Calculate the net torque $\vec{\tau}$, when the loop is at the position shown in the figure. **(5 pts)**

$$\vec{\tau} = \vec{\mu} \times \vec{B} = IA\hat{k} \times B\hat{j} = IAB(-\hat{i})$$ 3

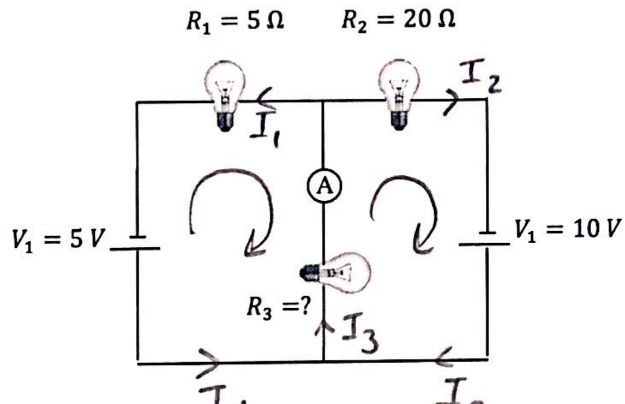
$$\vec{\tau} = -8 \times \frac{0.8\times0.6}{2}\,5\,\hat{i} = 9.6\,Am^2\,\hat{i}$$

2

3- a) In the circuit shown in the figure, the ammeter reads **0.4 A**. Find the resistance R_3 of the light bulb using Kirchhoff's Laws. **(10 pts)**

$I_3 = 0.4\,A$

$$\begin{cases} -5 + 5I_1 + I_3 R_3 = 0 \\ 10 - I_3 R_3 - 20 I_2 = 0 \\ I_1 + I_2 = I_3 = 0.4 \end{cases} \quad 4$$



$5 + 5I_1 - 20I_2 = 0$

$5 + 5(0.4 - I_2) - 20I_2 = 0$

$5 + 2 - 5I_2 - 20I_2 = 0$

$\frac{7}{25} = I_2$ 2 $\quad I_1 = 0.4 - \frac{7}{25} = \frac{3}{25}\,A$ 2

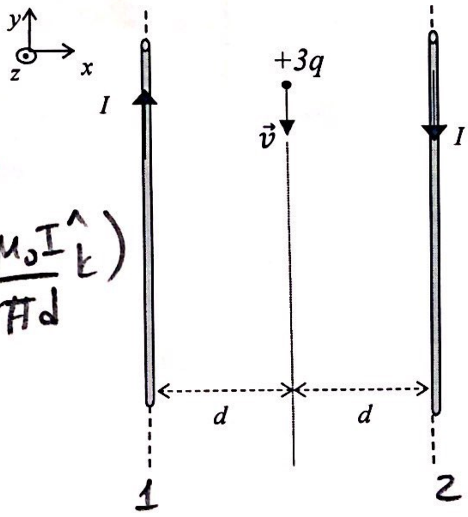
$-5 + 5\left(\frac{3}{25}\right) + 0.4R_3 = 0$

$-5 + \frac{3}{5} + 0.4R_3 = 0$ 2

$\frac{22}{5} = 0.4R_3 \Rightarrow R_3 = 11\,\Omega$

b) Two long parallel wires carry equal but opposite currents of I and separated by a distance $2d$. If you want to move a positive charge $+3q$ in the same plane and parallel to the wires as shown, write the <u>necessary electric field</u> $\vec{E}$ (both in magnitude and direction) that we have to apply. **(10 pts)**

$B_1 = \frac{\mu_0 I}{2\pi d}$, $B_2 = \frac{\mu_0 I}{2\pi d}$ 2



$\vec{B}_{net} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 I}{\pi d}(-\hat{k})$ 2

$\vec{F}_B = 3q\,\vec{v} \times \vec{B}_{net} = 3q(-v\hat{j}) \times \left(-\frac{\mu_0 I}{\pi d}\hat{k}\right)$

$\vec{F}_B = 3qv\mu_0 \frac{I}{\pi d}\hat{i}$ 2

$\vec{F}_{net} = \vec{F}_E + \vec{F}_B = 0$ 2

$3qv\mu_0 \frac{I}{\pi d}\hat{i} = -3q\vec{E}$

$\vec{E} = v\mu_0 \frac{I}{\pi d}(-\hat{i})$ 2

4- A rectangular wire loop, of sides a and b, is inside a region of magnetic field of $\vec{B} = \left(B_o \frac{x}{d}\right)(\hat{k})$, where B_o is an arbitrary positive constant, as shown in Figure 1.

a) Find the net magnetic flux Φ_B through the loop. **(7 pts)**

$$\Phi_B = \int \vec{B}\cdot d\vec{A} \quad 2$$

$$= \int_d^{d+a}\int_0^b B_o \frac{x}{d}\hat{k}\cdot dy\,dx\,\hat{k}$$

$$= B_o \left[\frac{x^2}{2d}\right]_d^{a+d}\left[y\right]_0^b = \frac{B_o b}{2d}\left[(a+d)^2 - d^2\right] \quad 5$$

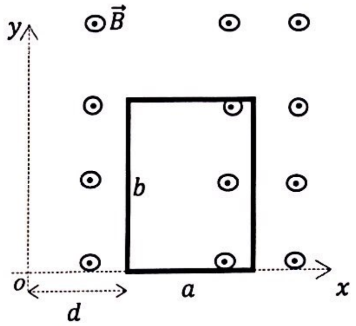


Figure 1

b) If the loop is rotating around its vertical axis with angular speed w as in Figure 2, calculate the induced emf ε_{ind} in it. **(7 pts)**

Hint: *the normal of the loop area makes an angle* $\boldsymbol{\theta = wt}$ *with the external magnetic field.*

$$\Phi_B = \int \vec{B}\cdot d\vec{A} = \frac{B_o b}{2d}\left[(a+d)^2 - d^2\right]\cos wt \quad 2$$

$$\varepsilon_{ind} = -\frac{d\Phi_B}{dt} = \frac{B_o b w}{2d}\left[(a+d)^2 - d^2\right]\sin wt \quad 5$$

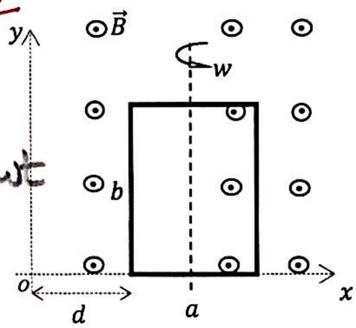


Figure 2

c) If the total resistance of the rotating loop is $\boldsymbol{R}$, determine the induced current $\boldsymbol{I_{ind}}$ in the loop and its direction when the loop is at the position shown in Figure 2. **(6 pts)**

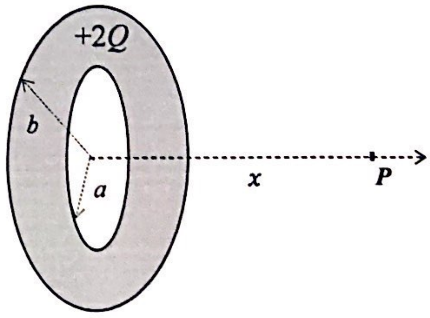
$$I_{ind} = \frac{\varepsilon_{ind}}{R} = \frac{B_o w b}{2dR}\left[(a+d)^2 - d^2\right]\sin wt$$

2 4

5- A disk with a hole at its center is shown in the figure. The disk has inner radius a and outer radius b and a net charge $2Q$ distributed uniformly over the surface.

a) Find the electric potential at point P on the x-axis. **(10 pts)**

Hint: *Electric potential of a ring of charge q and radius r is* $V = k\frac{q}{\sqrt{x^2+r^2}}$ *along its axis.*



$$dV = k\frac{dq}{(r^2+x^2)^{1/2}} \quad 2$$

$$dq = \sigma\, dA = \frac{2Q}{\pi(b^2-a^2)} 2\pi r\, dr \quad 2$$

$$dV = k\frac{4Qr\,dr}{(b^2-a^2)(r^2+x^2)^{1/2}} \Rightarrow V = \int dV = \frac{4kQ}{b^2-a^2}\int\frac{r\,dr}{(r^2+x^2)^{1/2}} \quad 2$$

$r^2+x^2 = u$

$2r\,dr = du$

$$V = \frac{4kQ}{b^2-a^2}\int\frac{du}{\underbrace{2u^{1/2}}_{2u^{1/2}}} = \frac{8kQ}{b^2-a^2}(r^2+x^2)^{1/2}\Big|_a^b$$

$$V = \frac{8kQ}{b^2-a^2}\left[(b^2+x^2)^{1/2} - (a^2+x^2)^{1/2}\right] \quad 4$$

b) Using your result from part a), find the electric field $\vec{E}$ at point P. **(10 pts)**

$$\vec{E} = -\frac{dV}{dx} = -\frac{8kQ}{b^2-a^2}\left[\frac{1}{2}(b^2+x^2)^{-1/2}2x - \frac{1}{2}(a^2+x^2)^{-1/2}2x\right] \quad 2$$

$$\vec{E} = -\frac{8kQx}{b^2-a^2}\left[\frac{1}{(b^2+x^2)^{1/2}} - \frac{1}{(a^2+x^2)^{1/2}}\right]$$

4 4